

2.9 Completing the Square

What we know when given equations in various forms

What can you tell from

standard form?

$$f(x) = ax^2 + bx + c$$

- direction of opening and stretch a
- the y intercept c



What can you tell from

Vertex form?

$$f(x) = a(x-h)^2 + k$$

- direction of opening and stretch a
- the vertex (h, k)
- max/min & when it occurs



⇒ find the zeros (set $= 0$ and solve for x using factored form)
 ⇒ y-intercept $(0, \quad)$

What can you tell from

Factored form?

$$f(x) = a(x-r)(x-s)$$

- direction of opening and stretch a
- the zeros r and s

As
Sub x to
get y



Algebraically...let's look for a pattern...

1. $(x+1)^2 = (x+1)(x+1) = x^2 + 2x + 1$

2. $(x-1)^2 = \underline{\hspace{2cm}} = x^2 - 2x + 1$

3. $\underline{\hspace{2cm}} = (x+2)(x+2) = x^2 + 4x + 4$

4. $(x-2)^2 = (x-2)(x-2) = \underline{\hspace{2cm}}$

5. $(x+\underline{\hspace{1cm}})^2 = \underline{\hspace{2cm}} = x^2 + 6x + \underline{\hspace{1cm}}$

6. $(x-\underline{\hspace{1cm}})^2 = \underline{\hspace{2cm}} = x^2 - 6x + \underline{\hspace{1cm}}$

7. $(x-\underline{\hspace{1cm}})^2 = \underline{\hspace{2cm}} = x^2 - 8x + \underline{\hspace{1cm}}$

So...to find the last term

divide the middle coefficient by 2 and square it

Let's try...

1. $x^2 + 10x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

2. $x^2 + 20x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

3. $y^2 - 12y + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

4. $m^2 - 3m + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

So factoring a perfect square $x^2 - 6x + 9 = (x-3)^2$  $\sqrt{\text{first term}}$ sign from
the middle $\sqrt{\text{last term}}$

OR
use half of
the middle
term

Complete the square

a) $x^2 - 10x + 1$

$$= x^2 - 10x + 25 - 25 + 1 \quad \left(\frac{-10}{2} = -5\right) \quad (-5)^2 = 25$$

$$= (x-5)^2 - 24$$

Vertex $(5, -24)$

b) $x^2 + 6x - 16$

$$= x^2 + 6x + 9 - 9 - 16 \quad \left(\frac{6}{2} = 3\right) \quad (3)^2 = 9$$

$$= (x+3)^2 - 25$$

Vertex $(-3, -25)$

c) $20x + x^2 + 3$

$$= x^2 + 20x + 3$$

$$= x^2 + 20x + 100 - 100 + 3$$

$$= (x+10)^2 - 97$$

$$\text{Vertex } (-10, -97)$$

What happens when there is a number in front of x^2

Complete the square

a) $5x^2 + 30x - 16$

$$\begin{aligned}
 &= 5(x^2 + 6x) - 16 \\
 &= 5(x^2 + 6x + 9 - 9) - 16 \\
 &= 5[(x+3)^2 - 9] - 16 \\
 &= 5(x+3)^2 - 45 - 16
 \end{aligned}$$

b) $-4.9x^2 + 29.4x + 7$

$$\begin{aligned}
 &= -4.9(x^2 - 6x) + 7 \\
 \text{ANS:} \\
 &= -4.9(x-3)^2 + 51.1
 \end{aligned}$$

c) $3x^2 + 12x$

$$\begin{aligned}
 &= 3(x^2 + 4x) \\
 &= 3(x^2 + 4x + 4 - 4) \\
 &= 3[(x+2)^2 - 4] \\
 &= 3(x+2)^2 - 12
 \end{aligned}$$

Would it be easier to find the vertex another way here?



Slightly more difficult...

Complete the square

a) $\frac{1}{4}x^2 + 5x + 2$

$$\begin{aligned}
 &= \frac{1}{4}(x^2 + 20x) + 2 \\
 &= \frac{1}{4}(x^2 + 20x + 100 - 100) + 2 \\
 &= \frac{1}{4}(x+10)^2 - 25 + 2
 \end{aligned}$$

b) $-3x^2 + 5x + 1$

$$\text{ANS} = -3\left(x - \frac{5}{6}\right)^2 + \frac{37}{12}$$

Hmwk:
Rest of Handout 2.9
& p 214 # 6 bce, 7b-f, 8, 9, 13, 17